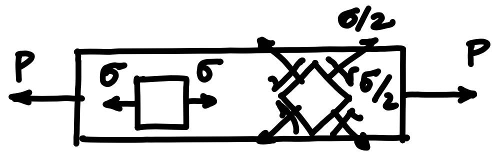
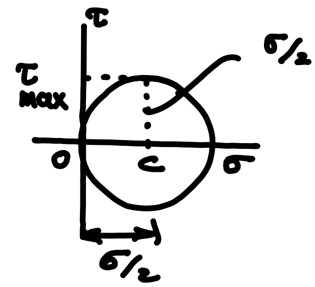
ME 314 - Engineering Design : Mechanical Components
Lecture 13

Note Title

Max Shear-Stress Theory (continued)

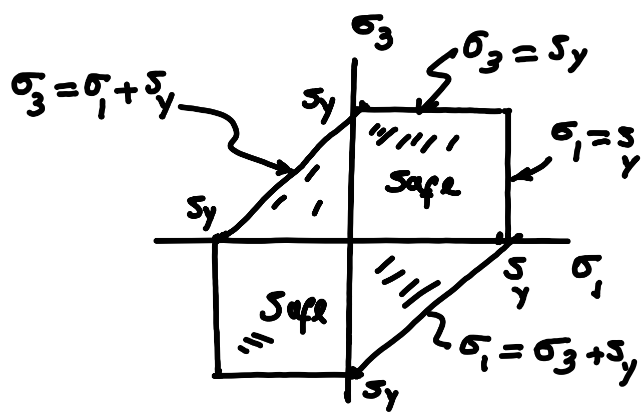
For tension test:





Consider a plane stress state for which $\sigma_2 = 0$:

1) If $\sigma_1 > \sigma_3 > 0$ then



2) If $\sigma_3 > \sigma_1 > 0$ then

MSS failure envelope is a hexagon

3) If $\sigma_1 < 0$, $\sigma_3 > 0$ then

4) If $\sigma_1 < \sigma_3 < 0$ then

5) If $\sigma_3 < \sigma_1 < 0$ then

6) If $\sigma_3 < 0$, $\sigma_1 > 0$ then

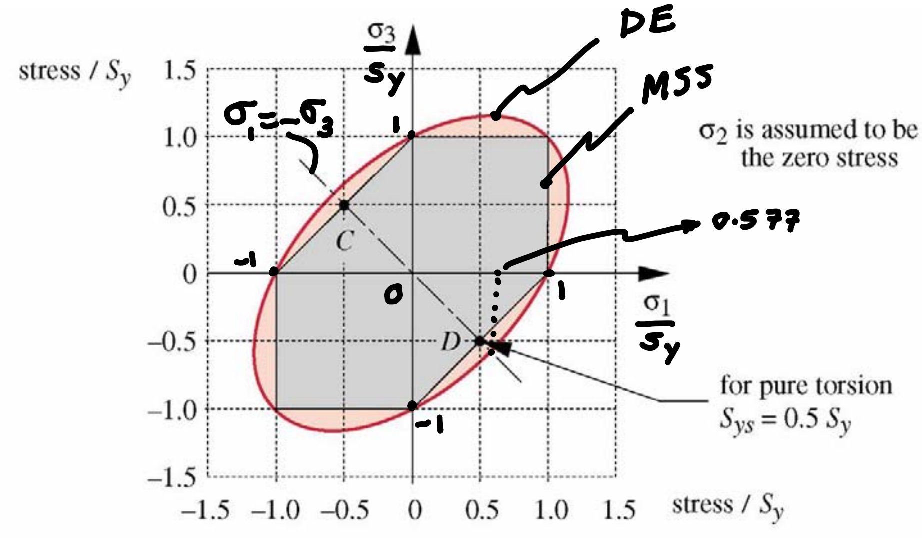
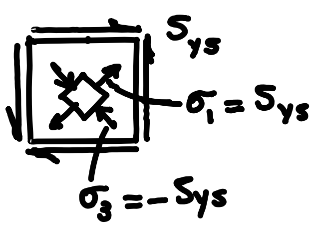


Figure 5-5

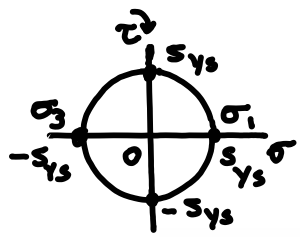
The 2-D Shear-Stress Theory Hexagon Inscribed Within the Distortion-Energy Ellipse.

For pure shear (torsion):



(This is at the intersection of "pure torsion" line and the MSS envelope in Fig. 5-5 above)

(This is the segment of MSS in the 4th- quadrant)



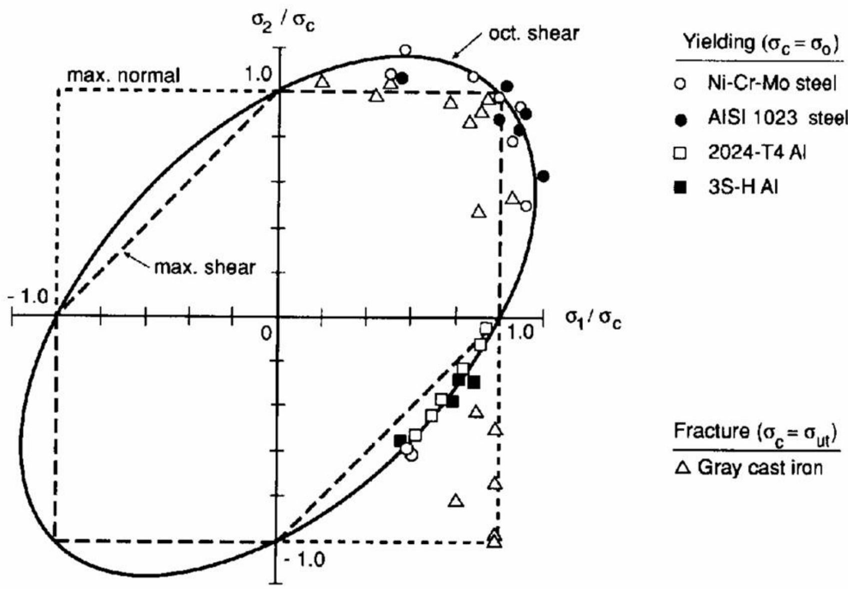
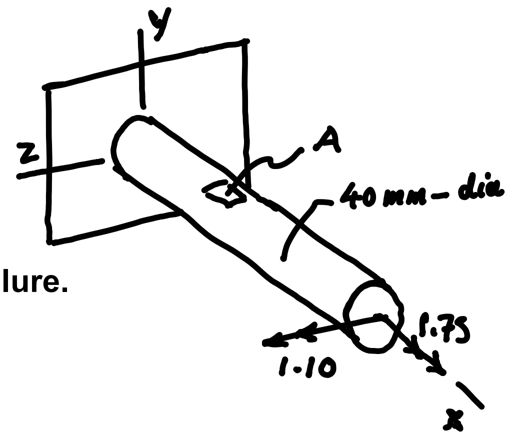


Figure 5-8

Experimental Data from Tensile Tests Superposed on Three Failure Theories *(Reproduced from Fig. 7.11, p. 252, in Mechanical Behavior of Materials by N. E. Dowling, Prentice-Hall, Englewood Cliffs, NJ, 1993)*.

Experimental data shows that both DE and MSS theories are acceptable in the case of static loading of ductile, homogeneous, isotropic materials whose compressive and tensile strength are equal (i.e., "even materials").



Example: The rod shown is made of a material with S_y = 345 MPa and is subjected to a couple:

$$M = 1.75\, i + 1.10\, k \qquad \text{kN-m}$$

Find the a factor of safety guarding against a static of failure.

5.2 Failure of Brittle Materials under Static Loading

Brittle materials fracture rather than yield. Some brittle materials (e. g., wrought materials such as fully hardened steel) are among the so-called "even materials" which tend to have equal compressive and tensile strengths. Many brittle materials (e.g., cast materials such as gray cast iron), however, have compressive strength that are much greater than their tensile strength. These are called "uneven materials."

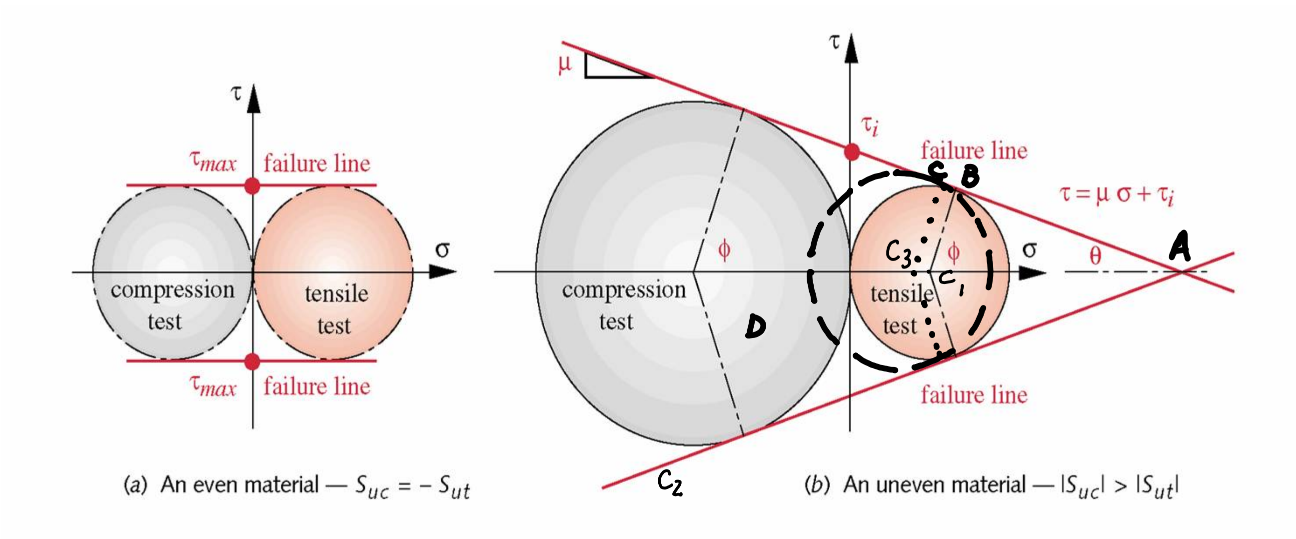


Figure 5-10

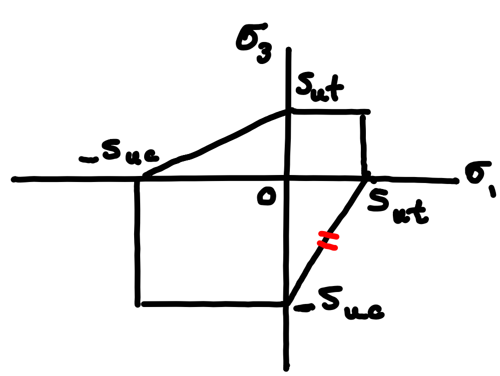
Mohr's Circles for Both Compression and Tensile Tests Showing the Failure Envelopes for *(a) Even and (b) Uneven Materials.*

The above figure shows Mohr's circles for both compression and tension tests of an even (a) and an uneven (b) material. The lines tangent to these circles are failure lines for all applied stresses. The area enclosed by the circles and the failure lines represent a safe zone.

For even materials ($S_{ut} = S_{uc}$), the failure lines do not depend on the normal stress σ. They only depend on the shear strength of the material. This is consistent with the MSS theory for ductile materials which are also even ($S_{ut} = S_{uc}$).

For uneven materials ($S_{ut} \neq S_{uc}$), the failure lines are functions of both σ and τ when one principal stress is tensile and the other is compressive. The appropriate failure criteria in this case is called the Coulomb-Mohr theory as shown in Figure below.

However, the actual failure data do not follow the Coulomb-Mohr theory in the 4th-quadrant where $\sigma_1 \geq 0 \geq \sigma_3$. Hence, the so-called Modified-Mohr theory is used.



$\sigma_{ut} \equiv S_{ut}$

$\sigma_{uc} \equiv S_{uc}$

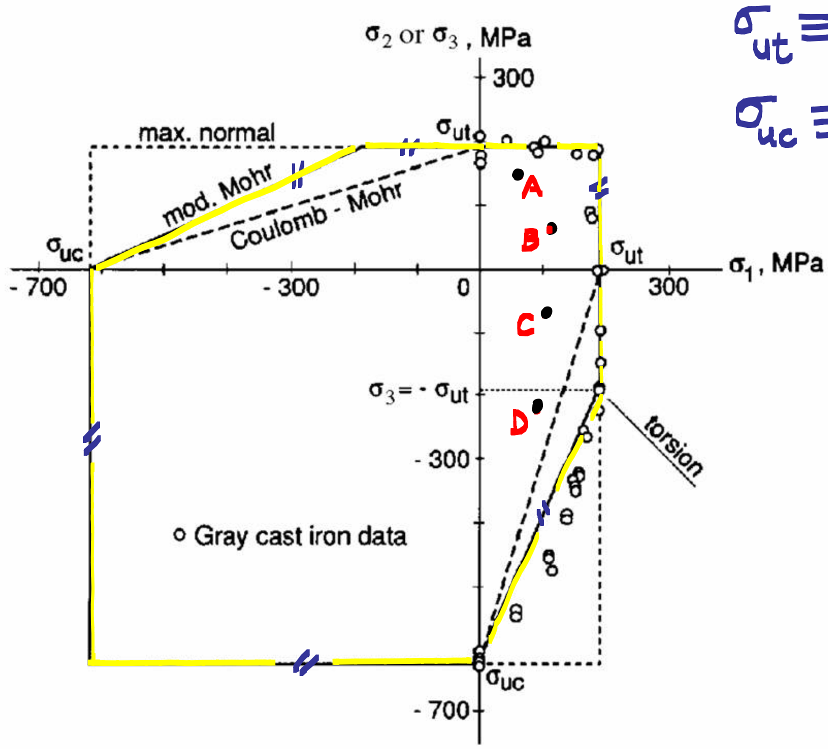
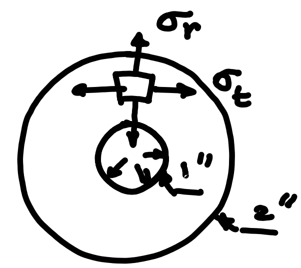


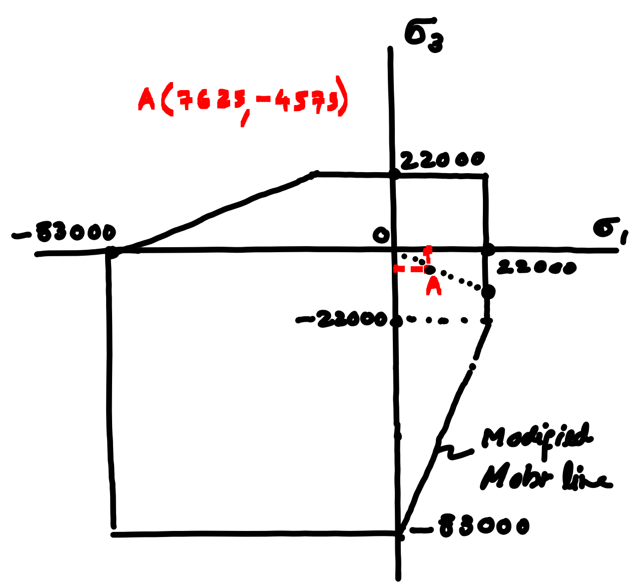
Figure 5-12

Biaxial Fracture Data of Gray Cast Iron Compared to Various Failure Criteria *(From Fig 7.13, p. 255, inMechanical Behavior of Materials by N. E. Dowling, Prentice-Hall, Englewood Cliffs, NJ, 1993. Data from R. C. Grassi and I. Cornet,"Fracture of Gray Cast Iron Tubes under Biaxial Stresses," J. App. Mech, v. 16, p.178, 1949)* .

To apply this criterion, we first obtain the principal stresses σ_1 and σ_3 for the problem of interest and plot the point (σ_1 , σ_3) in the σ_1-σ_3 plane shown above. Points A, B, C, and D are shown as examples.

Example: A thick-walled cylinder is made of ASTM grade 20 cast iron (S_{ut} = 22 kpsi, S_{uc} = 83 kpsi) and it has an internal radius of 1 in and an outer radius of 2 in. Find the factor of safety for guarding against failure if the cylinder is under an internal pressure of 4575 psi.

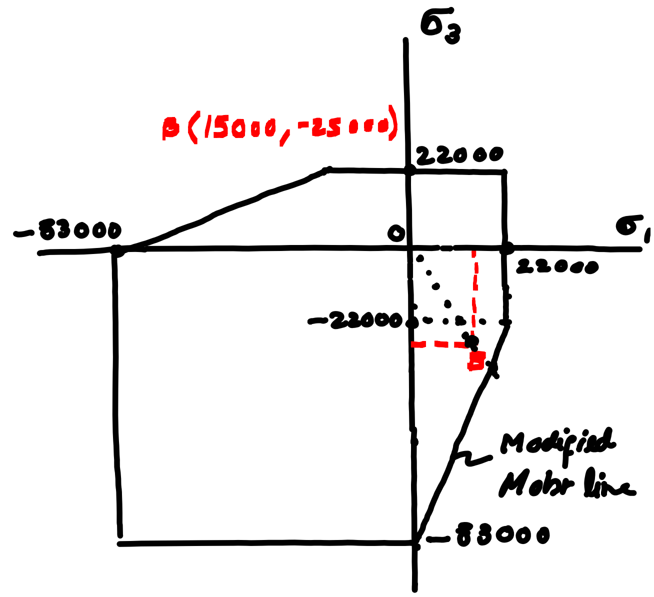




For the sake of example, suppose we had found that:

σ_1 = 15,000 psi
σ_3 = - 25,000 psi

This will correspond to point B on the graph. In this case, OB intersects the Modified-Mohr line and the factor of safety is calculated from Eq. (5.12b):



There is an alternative approach (proposed by Dowling) for using the Modified-Mohr failure theory that does not require drawing the diagram. In this approach, we define a modified-Mohr effective stress, $\tilde{\sigma}$:

$$\left.\begin{aligned} &\tilde{\sigma} = MAX\,(C_1, C_2, C_3, \sigma_1, \sigma_2, \sigma_3) \\ &\tilde{\sigma} = 0 \quad \text{if} \quad MAX < 0 \end{aligned}\right\} \quad (5.12d)$$

where the function MAX denotes the algebraically largest of the six arguments. If all arguments are negative, then the effective stress is zero. In Eq. (5.12d), the arguments C_l, C_2, and C_3 are given by:

$$\left.\begin{aligned} C_1 &= \frac{1}{2}\left[|\sigma_1 - \sigma_2| + \frac{2S_{ut} - S_{uc}}{-S_{uc}}(\sigma_1 + \sigma_2)\right] \\ C_2 &= \frac{1}{2}\left[|\sigma_2 - \sigma_3| + \frac{2S_{ut} - S_{uc}}{-S_{uc}}(\sigma_2 + \sigma_3)\right] \\ C_3 &= \frac{1}{2}\left[|\sigma_3 - \sigma_1| + \frac{2S_{ut} - S_{uc}}{-S_{uc}}(\sigma_3 + \sigma_1)\right] \end{aligned}\right\} \quad (5.12c)$$

The modified-Mohr effective stress, σ is then compared to the ultimate tensile strength, S_{ut}:

For the example we just solved,

σ_1=7625, σ_2 = 0, σ_3 = - 4573, S_{ut} = 22000, S_{uc} = 83000 psi

Since

$$\frac{2S_{ut} - S_{uc}}{-S_{uc}} = \frac{2(22000) - 83000}{-83000} = 0.47 \quad ,$$

we have

$$C_1 = \frac{1}{2}\left[\,|7625 - 0| + 0.47\,(7625)\right] = 5604$$

$$C_2 = \frac{1}{2}\left[\,|0 + 4575| + 0.47(0 - 4575)\right] = 1213$$

$$C_3 = \frac{1}{2}\left[\,|-4575 - 7625| + 0.47(-4575 + 7625)\right] = -5383$$

$$\tilde{\sigma} = MAX\left[5604,\ 1213,\ -5383,\ 7625,\ 0,\ -4575\right] = 7625$$

$$\therefore\ N = \frac{S_{ut}}{\tilde{\sigma}} = \frac{22000}{7625} = 2.89 \qquad \triangleleft$$

as found earlier. Also, for the hypothetical case that

σ_1 = 15000, σ_2 = 0, σ_3 = - 25000, S_{ut} = 22000, S_{uc} = 83000 psi

we find C_l, C_2, and C_3:

C_l = 11025, C_2 = 6625, C_3 = 17630

and the modified-Mohr effective stress:

$$\tilde{\sigma} = MAX\left[11025,\ 6625,\ 17650,\ 15000,\ 0,\ -25000\right]$$

$$= 17650 \quad psi$$

Hence,

$$N = \frac{S_{ut}}{\tilde{\sigma}} = \frac{22000}{17650} = 1.25 \qquad \triangleleft$$

as found earlier.